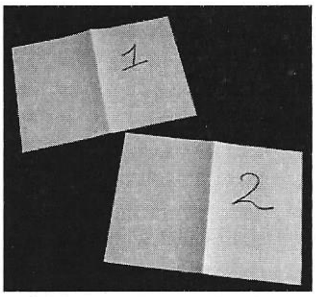
Exam NANP1-10 Physics Laboratory 1: Data and error analysis
10 November 2016 (09:00 - 11:00)

Please note:

- **DO NOT OPEN THE EXAM BEFORE YOU ARE TOLD TO DO SO**
- This exam consists of **5 exercises** on 5 pages.
- Make each exercise on a separate sheet of paper!



- Write your name and student number on each sheet of paper!
- Write clearly, using a pen (not a pencil).
- A simple scientific calculator is allowed during the exam, but a graphing calculator is not permitted.
- When finished, hand in these exercises together with your own answers.

Points per exercise

Exercise		total	Exercise		total
1a,b,c,d	1 each		4a	1	
1		4	4b	2	
2a	1		4c	2	
2b	1		4d	2	
2c	2		4e	2	
2d	1		4		9
2e	4		5a	2	
2f	1		5b	2	
2		10	5c	4	
3a	2		5d	3	
3b	2		5e	2	
3		4	5		13

Exam mark = *(total of points / 4.5)* + $\frac{10}{9}$

Exercise 1 *(4 points)* **Use a new sheet of paper** (see photo)

Rewrite the following results, using the correct notation (proper accuracy and number of significant digits):

a) $p = 2101.325$ kPa $\pm$ 240 Pa

b) $\lambda = 0.589631$ μm $\pm$ 0.078 nm

c) $T = 4.187$ K $\pm$ 81 μK

d) $R = 5618$ kΩ $\pm$ 0.21 MΩ

Exercise 2 *(10 points)* **Use a new sheet of paper** (see photo)

The phase velocity c for longitudinal waves ("the speed of sound c") in liquids is given by:

$$c = \sqrt{K/\rho}$$

with K the compression modulus and ρ the density of the liquid. The metal mercury is liquid at room temperature and in an experiment a long reservoir filled with mercury is used to determine the speed of sound and the compression modulus of mercury.

a) **Give** the (SI) units of K. Show at least one intermediate step.

The length of the reservoir is $L = 10.00 \pm 0.01$ m. A sound is generated at one end of the reservoir at $t_0 = 0$ s, which is received at the other end at $t_1 = 6.85$ s. The relative error of t_1 is 1% (the error in t_0 is negligible).

b) **Calculate** the speed of sound c in mercury.

c) **Calculate** the relative error of c.

If you do not have an answer to part b) and c), use $c = 2580 \pm 13$ m s^{-1} for the rest of the exercise. The density of mercury is $\rho = (13.5 \pm 0.4) \cdot 10^3$ kg m^{-3}.

d) **Calculate** the compression modulus K.

e) **Calculate** the relative and absolute error in K.

f) **Write** the final result in the correct notation: $K = \ldots \pm \ldots$ ...

Exercise 3 *(4 points)* **Use a new sheet of paper** (see photo)

The refractive index n of diamond is determined with 3 independent techniques. The results are: $n_1 = 2.417 \pm 0.001$, $n_1 = 2.415 \pm 0.003$, $n_1 = 2.421 \pm 0.005$.

a) **Calculate** the weighted average refractive index n_{avg}.

b) **Calculate** the error Δn of n_{avg}.

Exercise 4 *(9 points)* **Use a new sheet of paper** (see photo)

Radon is a radioactive noble gas. The decay of a sample of radon is measured and an *average* rate of 3.0 decays per second is observed.

a) **Multiple choice question:** what is the probability P of observing *exactly* 3 decays in 1 second?
A: $P = 100\%$
B: $P = 50\%$
C: $P = 22\%$
D: $P = 33.3\%$

Your answer does not have to contain more than a single letter {A, B, C, D}. Any arguments or calculations given will be ignored. Given:

$$f(k) = \frac{\lambda^k}{k!}e^{-\lambda}$$

Now the density ρ of radon is determined 5 times. The results are: $\rho_1 = 9.73$, $\rho_2 = 9.68$, $\rho_3 = 9.77$, $\rho_4 = 9.72$, $\rho_5 = 9.70$ kg m^{-3}.

b) **Calculate** the best estimate for the average density.

c) **Calculate** the best estimate s for the standard deviation of these measurements.

d) **Calculate** the error s_m in the best estimate for the average density calculated in part a).

A very large number of measurements has now been used to determine the density. The measurement results follow a Gaussian distribution, with an average $\rho_{av} = 9.730$ kg m^{-3} and a standard deviation $\sigma_\rho = 0.025$ kg m^{-3}.

e) **Calculate** the probability of finding an individual measurement result (of the very large number of measurements) in the range $9.710 \leq \rho \leq 9.770$ kg m^{-3}.

If necessary, use table 2 (on the last page) and:

$$N(y) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{y^2}{2}\right) \qquad \text{and} \qquad F(z) = \int_0^z N(y)dy$$

Exercise 5 *(13 points)* **Use a new sheet of paper** (see photo)

x	$y \pm \Delta y$
-3	6 ± 1
-1	4 ± 2
2	-2 ± 2
4	-3 ± 1

Table 1: Observations for exercise 5.

A series of 4 observations is given in table 1 above. The error in x is negligible. A straight line $y = ax + b$ is fitted to these observations.

The following formulae are given for fitting data to a straight line $y = ax + b$:

$$a = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2},$$

$$b = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2},$$

$$\Delta a = \sqrt{\left(\frac{1}{\sum x_i^2 - N\overline{x}^2}\right)\frac{\sum r_i^2}{N-2}},$$

$$\Delta b = \sqrt{\left(\frac{1}{N} + \frac{\overline{x}^2}{\sum x_i^2 - N\overline{x}^2}\right)\frac{\sum r_i^2}{N-2}}.$$

a) **Calculate** the best estimate for a and b using the method of least squares.

If you do not have the answer to part a), use $a = -2$ and $b = 3$ for the rest of the exercise.

b) **Calculate** the errors in a and b.

If you do not have the answer to part b), use $\Delta a = 0.9$ and $\Delta b = 1.1$ for the rest of the exercise.

c) **Calculate** y and Δy for a value of $x = 1$.

The student who has carried out the experiment wants to use the chi-square test to check whether the linear fit is acceptable. She decides to use the 10% - 90% acceptance requirement.

d) **Calculate** χ^2.

e) **Indicate** whether the linear fit is acceptable or not. If necessary, use table 3 (on the last page).

Tables

z	$F(z)$	z	$F(z)$	z	$F(z)$	z	$F(z)$
0.0	0.0000	1.0	0.3413	2.0	0.4772	3.0	0.4987
0.1	0.0398	1.1	0.3643	2.1	0.4821	3.1	0.4990
0.2	0.0793	1.2	0.3849	2.2	0.4861	3.2	0.4993
0.3	0.1179	1.3	0.4032	2.3	0.4893	3.3	0.4995
0.4	0.1554	1.4	0.4192	2.4	0.4918	3.4	0.4997
0.5	0.1915	1.5	0.4332	2.5	0.4938	3.5	0.4998
0.6	0.2258	1.6	0.4452	2.6	0.4953	3.6	0.4998
0.7	0.2580	1.7	0.4554	2.7	0.4965	3.7	0.4999
0.8	0.2881	1.8	0.4641	2.8	0.4974	3.8	0.4999
0.9	0.3159	1.9	0.4713	2.9	0.4981	3.9	0.5000

Table 2: Numerical values of the Gaussian integral function $F(z)$.

$F =$ ν	0.01	0.10	0.50	0.90	0.99
1	0.000	0.016	0.455	2.706	6.635
2	0.020	0.211	1.386	4.605	9.210
3	0.115	0.584	2.366	6.251	11.35
4	0.297	1.064	3.357	7.779	13.28
5	0.554	1.610	4.351	9.236	15.09

Table 3: Cumulative χ^2 distribution $F(\chi^2|\nu)$.